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Version of attached file:

Published Version

Peer-review status of attached file:

Peer-reviewed

Citation for published item:

Englert, Christoph and Re, Emanuele and Spannowsky, Michael (2013) 'Pinning down Higgs triplets at the LHC.', Physical review D., 88 (3). 035024.

Further information on publisher's website:

<http://dx.doi.org/10.1103/PhysRevD.88.035024>

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Pinning down Higgs triplets at the LHCChristoph Englert,^{1,2,*} Emanuele Re,^{3,†} and Michael Spannowsky^{1,‡}¹*Department of Physics, Institute for Particle Phenomenology, Durham University, Durham DH1 3LE, United Kingdom*²*SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*³*Department of Physics, Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, United Kingdom*

(Received 10 July 2013; published 28 August 2013)

Extensions of the standard model Higgs sector involving weak isotriplet scalars are not only benchmark candidates to reconcile observed anomalies of the recently discovered Higgs-like particle, but also exhibit a vast parameter space, for which the lightest Higgs' phenomenology turns out to be very similar to the standard model one. A generic prediction of this model class is the appearance of exotic doubly charged scalar particles. In this paper we adapt existing dilepton + missing energy + jets measurements in the context of supersymmetry searches to the dominant decay mode $H^{\pm\pm} \rightarrow W^\pm W^\pm$ and find that the LHC already starts probing the model's parameter space. A simple modification toward signatures typical of weak boson fusion searches allows us to formulate even tighter constraints with the 7 TeV LHC data set. A corresponding analysis of this channel performed at 14 TeV center-of-mass energy will constrain the model over the entire parameter space and facilitate potential $H^{\pm\pm} \rightarrow W^\pm W^\pm$ discoveries.

DOI: [10.1103/PhysRevD.88.035024](https://doi.org/10.1103/PhysRevD.88.035024)

PACS numbers: 12.60.Fr, 14.80.Fd, 13.85.Rm

I. INTRODUCTION

The recent discovery [1–4] of the Higgs boson [5] provides an opportunity to check the phenomenological consistency of various scenarios of electroweak symmetry breaking with measurements for the first time. Higgs triplet models have received considerable attention recently as they can reconcile the possibly observed anomaly in the $H \rightarrow \gamma\gamma$ channel [6–9]. Whether this excess persists or future measurements of the diphoton partial decay width will return to the standard model (SM) values as suggested by recent CMS results [10] is unclear at the moment. However, as demonstrated in [8], there are certain models with Higgs triplets [11,12] that possess a large parameter space where the resulting phenomenology is SM-like [13,14] even for larger triplet vacuum expectation values. A generic prediction of electroweak precision measurements in this case is the appearance of doubly charged scalar particles $H^{\pm\pm}$ with a mass of several hundred GeV that result from the weak triplet structure in the Higgs sector extension.

Because of the quantum numbers of the $SU(2)_L$ triplet, Majorana mass-type operators can induce a prompt decay of $H^{\pm\pm}$ into two leptons with identical charge [15]. This interaction has already been constrained at the LHC in multilepton searches [16]. However, as soon as the mass of the doubly charged scalar exceeds twice the W mass, the decay to gauge bosons is preferred. This can be seen from the scaling of the partial decay widths:

$\Gamma(H^{\pm\pm} \rightarrow W^\pm W^\pm)/\Gamma(H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm) \sim m_{H^{\pm\pm}}^2/m_W^2$. Over the bulk of the parameter space this leads to a dominant decay of the doubly charged Higgs to W bosons [17]. Formulating a meaningful constraint of this model class must therefore not neglect $H^{\pm\pm} \rightarrow W^\pm W^\pm$ [18].

The production of the single intermediate $H^{\pm\pm}$ boson can only proceed via weak boson fusion (WBF) diagrams (Fig. 1) and crossed processes (i.e. Drell-Yan-type production). Hence, $H^{\pm\pm}$ production inherits all the phenomenological advantages of WBF Higgs and diboson production [19]. Producing the relatively heavy final state requires energetic initial state partons. The t -channel color singlet exchange results in relatively small scattering angles of the two outgoing jets at moderate transverse momentum and a central detector region essentially free of QCD radiation. Eventually, the typical signature is two isolated central leptons and missing energy, and two forward jets at large rapidity differences with high invariant mass. Phenomenological investigations of these signatures are helped by small irreducible SM backgrounds [20,21]. These signatures have already been investigated partially in Refs. [22,23], however, including neither a parameter scan involving the Higgs candidate's signal strengths nor constraints from electroweak precision data (EWPD).

To our knowledge, neither ATLAS nor CMS have performed a dedicated analysis of this final state in the triplet Higgs model context. However, there are searches for supersymmetry (SUSY) in same-sign dilepton events with jets and missing energy [24,25], where the same-sign leptons arise from the decay chains of the pair-produced gluino or squark particles' cascade decays [26]. Such a process is mediated by a nontrivial color exchange in the s or t channels, which results in large scattering

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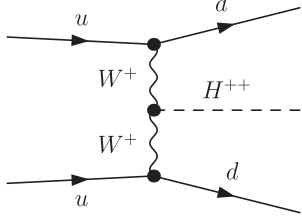


FIG. 1. Sample weak boson fusion diagram involved in the production of $H^{\pm\pm}$. We do not show the $H^{\pm\pm}$ decay. By crossing one of the up-flavor quarks to the final state and the non-connected down-flavor quark to the initial state we recover the Drell-Yan-type production modes.

angles of the energetic final state jets. This signature, characterized by large $H_T = \sum_{i \in \text{jets}} p_{T,i}$, is different from the typical WBF phenomenology. On the other hand, since $\text{Br}(H^{\pm\pm} \rightarrow W^\pm W^\pm)$ is large, we might overcome the limitations of searches for light Higgs particles in $H \rightarrow VV$, $V = Z, W^\pm$, especially because the $H^{\pm\pm} W^\mp W^\mp$ coupling can be enhanced in comparison to HW^+W^- due to the model's triplet character. Furthermore, Ref. [25], which reports a SUSY search employing the 7 TeV 4.98 fb $^{-1}$ data set, comprises signal regions with relatively small $H_T \geq 80$ GeV (compensated with a larger missing energy requirement) that can be exploited to formulate constraints on the triplet model. This will be the focus of Sec. III. Subsequently, in Sec. III A, we demonstrate that a slight modification of the search strategy of Ref. [25] is sufficient to obtain superior constraints on the triplet model even for a pessimistic estimate of reducible backgrounds and other uncertainties. We also discuss how far these estimates can be improved by including the 8 TeV data set. In Sec. IV we discuss an analysis on the basis of a WBF selection at $\sqrt{s} = 14$ TeV center-of-mass energy, which will yield strong constraints on the triplet models' parameter space.

As we will argue, the results of these sections are not specific to a particular triplet model and largely generalize to *any* model with Higgs triplets. Since the tree-level custodial symmetry preserving implementation of Higgs triplets exhibits a richer phenomenology, we specifically analyze the impact of the described searches in the context of the Georgi-Machacek (GM) model [11] (which we quickly review in Sec. II to make this work self-contained). In particular, we input the direct search constraints for doubly charged scalars into a global scan of the electroweak properties, also taking into account EWPD. We give our summary in Sec. VI.

II. A CONSISTENT MODEL OF HIGGS TRIPLETS

The Georgi-Machacek model [11] is a tree-level custodial isospin-conserving implementation of Higgs triplets based on scalar content

$$\Phi = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}, \quad \Xi = \begin{pmatrix} \chi_3^* & \xi_1 & \chi_1 \\ -\chi_2^* & \xi_2 & \chi_2 \\ \chi_1^* & -\xi_1^* & \chi_3 \end{pmatrix}. \quad (1)$$

Φ is a SM-like Higgs doublet necessary for introducing fermion masses, and Ξ combines the complex (χ_1, χ_2, χ_3) and real $(\xi_1, \xi_2, -\xi_1^*)$ triplets such that an additional $SU(2)_R$ can act in the usual fashion ($\Xi \rightarrow U_L \Xi U_R^\dagger$ and $\Phi \rightarrow \tilde{U}_L \Phi \tilde{U}_R^\dagger$) leaving custodial isospin unbroken after Φ and Ξ obtain vacuum expectation values (vevs) $\langle \Xi \rangle = v_\Xi \mathbb{1}$, $\langle \Phi \rangle = v_\Phi \mathbb{1}$.

For the purpose of this paper we choose a Higgs sector Lagrangian

$$\mathcal{L} = \frac{1}{2} \text{Tr}[D_{2,\mu} \Phi^\dagger D_2^\mu \Phi] + \frac{1}{2} \text{Tr}[D_{3,\mu} \Xi^\dagger D_3^\mu \Xi] - V(\Phi, \Xi) + \Phi \text{ Yukawa interactions}, \quad (2a)$$

where we introduce the potential that triggers electroweak symmetry breaking

$$V(\Phi, \Xi) = \frac{\mu_2^2}{2} \text{Tr}(\Phi^c \Phi) + \frac{\mu_3^2}{2} \text{Tr}(\Xi^c \Xi) + \lambda_1 [\text{Tr}(\Phi^c \Phi)]^2 + \lambda_2 \text{Tr}(\Phi^c \Phi) \text{Tr}(\Xi^c \Xi) + \lambda_3 \text{Tr}(\Xi^c \Xi \Xi^c \Xi) + \lambda_4 [\text{Tr}(\Xi^c \Xi)]^2 - \lambda_5 \text{Tr}(\Phi^c t_2^a \Phi t_2^b) \text{Tr}(\Xi^c t_3^a \Xi t_3^b). \quad (2b)$$

This choice reflects the properties of the Higgs triplet model in a simplified way [11] and can be motivated from imposing a \mathbb{Z}_2 symmetry [12].

D_2, D_3 are the gauge-covariant derivatives in the $SU(2)_L$ doublet and triplet representations. Hypercharge $U(1)_Y$ is embedded into $SU(2)_R$ as in the SM, the $\mathfrak{su}(2)$ generators in the triplet representation are

$$t_3^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_3^2 = \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (3)$$

$$t_3^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

The masses of the electroweak bosons m_W, m_Z after symmetry breaking follow from the sum of the Higgs fields' vevs, constraining

$$(246 \text{ GeV})^2 = v_\Phi^2 + 8v_\Xi^2. \quad (4)$$

Defining the mixing angles

$$\cos \theta_H =: c_H = \frac{v_\Phi}{v_{\text{SM}}}, \quad \sin \theta_H =: s_H = \frac{2\sqrt{2}v_\Xi}{v_{\text{SM}}} \quad (5)$$

turns out to be useful. Since custodial isospin is preserved, in the unitary gauge the Higgs masses group into two singlets, one triplet and one quintet (the quintet includes

our doubly charge scalar $H_5^{\pm\pm}$, which we will indicate also without the subscript). Their masses are

$$\begin{aligned} m_{H_0}^2 &= 2(2\lambda_1 v_\Phi^2 + 2(\lambda_3 + 3\lambda_4)v_\Xi^2 + m_{\Phi\Xi}^2), \\ m_{H'_0}^2 &= 2(2\lambda_1 v_\Phi^2 + 2(\lambda_3 + 3\lambda_4)v_\Xi^2 - m_{\Phi\Xi}^2), \\ m_{H_3}^2 &= \frac{1}{2}\lambda_5(v_\Phi^2 + 8v_\Xi^2), \\ m_{H_5}^2 &= \frac{3}{2}\lambda_5 v_\Phi^2 + 8\lambda_3 v_\Xi^2, \end{aligned} \quad (6)$$

with shorthand notation

$$\begin{aligned} m_{\Phi\Xi}^2 &= [4\lambda_1^2 v_\Phi^4 - 8\lambda_1(\lambda_3 + 3\lambda_4)v_\Phi^2 v_\Xi^2 \\ &\quad + v_\Xi^2(3(2\lambda_2 - \lambda_5)^2 v_\Phi^2 + 4(\lambda_3 + 3\lambda_4)^2 v_\Xi^2)]^{1/2}. \end{aligned} \quad (7)$$

To reach Eq. (6) we have diagonalized the singlet mixing by an additional rotation

$$H_0 = c_q H_\Phi + s_q H_\Xi, \quad H'_0 = -s_q H_\Phi + c_q H_\Xi, \quad (8)$$

with angle

$$\sin \angle(H_\Phi, H_0) =: s_q = \frac{\sqrt{3}}{\sqrt{3 + \left[\frac{2\lambda_1 v_\Phi^2 - 2(\lambda_3 + 3\lambda_4)v_\Xi^2 + m_{\Phi\Xi}^2}{(2\lambda_2 - \lambda_5)v_\Phi v_\Xi} \right]^2}}. \quad (9)$$

Note that $m_{H'_0} < m_{H_0}$, and therefore $m_{H'_0}$ will be the observed Higgs boson.

We straightforwardly compute the couplings of the uncharged states to the SM fermions f and gauge bosons v , normalized to the SM expectation, as

$$\begin{aligned} c_{f,H_0} &= \frac{c_q}{c_H}, \\ c_{v,H_0} &= c_q c_H + \sqrt{8/3} s_q s_H, \\ c_{f,H'_0} &= -\frac{s_q}{c_H}, \\ c_{v,H'_0} &= -s_q c_H + \sqrt{8/3} c_q s_H. \end{aligned} \quad (10)$$

The custodial triplet (H_3^+, H_3^0, H_3^-) is gaugephobic and the quintet fermiophobic with the additional assumption of a vanishing leptonic Majorana operator. For the purpose of our analysis this does not pose any phenomenological restriction. Since $\langle \Xi \rangle$ is the order parameter that measures the degree of triplet symmetry breaking, a measurement of the $H^{\pm\pm} \rightarrow W^\pm W^\pm$ directly reflects the phenomenology's triplet character. Indeed, the vertex we are predominantly interested in is given by

$$H^{\pm\pm} W_\mu^\mp W_\nu^\mp: \sqrt{2} i g m_W s_H g_{\mu\nu}, \quad (11)$$

and, as we mentioned in Sec. I, the relevant final states to study this vertex are therefore “ $E_T^{\text{miss}} + \ell^\pm \ell^\pm$ ” in

association with at least 2 jets. The 2 jets signature will play the more important role.

Note that Eq. (11) implies that $H^{\pm\pm}$ can be enhanced by up to a factor of 2 compared to the WBF production of a neutral SM-like Higgs boson of the same mass. The enhanced couplings Eqs. (10) and (11) are a direct consequence of the larger isospin of the triplet that feeds into the interactions via the gauge kinetic terms.

At this stage it is important to comment on the relation of the Georgi-Machacek model with “ordinary” triplet Higgs extension, e.g. when we just add a complex scalar field to the SM Higgs sector with hypercharge $Y = 2$ [15]. Such models introduce a tree-level custodial isospin violation, and consistency with EWPD imposes a hierarchy of the vevs ($s_H \ll 1$). Since we are forced to tune the model already at tree level, the additional singly and doubly charged states tend to decouple from the phenomenology apart from loop-induced effects on branching ratios (see e.g. [7] for reconciling the possibly observed excess in $H \rightarrow \gamma\gamma$ in this fashion). The Georgi-Machacek model is *fundamentally* different in this respect: because of the $SU(2)_R$ invariant extension of the Higgs potential there are no tree-level constraints on v_Ξ . In fact, only the generation of fermion masses requires the presence of another doublet, and $2\sqrt{2}v_\Xi \gg v_\Phi$ does not lead to tree-level inconsistencies in the gauge sector. At one loop, however, this picture changes. The presence of a triplet requires the explicit breaking of $SU(2)_R$ invariance to tune the ρ parameter to the values consistent with EWPD [8,27], but still larger values of v_Ξ remain allowed in comparison to the simple complex triplet extension, where recent upper bounds for the triplet vev read as $v_{\text{triplet}} < 0.03 \times (246 \text{ GeV})$ [7].

An analysis that measures $H_5^{\pm\pm} \rightarrow W^\pm W^\pm$ is not specific to the underlying model as Eq. (11) simply follows from the presence of a triplet Higgs in the particle spectrum that contributes to electroweak symmetry breaking. Since the Georgi-Machacek model accommodates larger values of s_H with a rich phenomenology, we take this particular model as a benchmark for our parameter fit in Sec. V. Our results generalize to any triplet Higgs model implementation—they provide constraints on this branching ratio, which are model-independent statements as long as the narrow width approximation can be justified.

III. REINTERPRETING SUSY SEARCHES

We are now ready to compute an estimate of the performance of the CMS analysis of Ref. [25] when reinterpreted in the Higgs triplet context.

We focus on the light lepton flavor channel of Ref. [25]; the additional τ lepton channels are subject to large fake background uncertainties and do not provide statistical pull for our scenario in the first place. The CMS analysis of Ref. [25] clusters anti- k_T jets [28] with $R = 0.5$ as implemented in FASTJET [29] and selects jets with $p_T > 40 \text{ GeV}$

in $|\eta| < 2.5$. Leptons are considered as isolated objects if the hadronic energy deposit within $\Delta R = [(\Delta\phi)^2 + (\Delta\eta)^2]^{1/2} = 0.3$ is less than 15% of the lepton candidate's p_T . The thresholds are $p_{T,\mu} > 5$ GeV, and $p_{T,e} > 10$ GeV, and there is a “high p_T ” selection with $p_{T,\ell} > 10$ GeV ($\ell = e, \mu$) with the hardest lepton having $p_T > 20$ GeV. All leptons need to fall within $|\eta| < 2.4$. CMS requires at least two jets and two leptons and vetoes events with three leptons when one of the leptons combines with one of the others to the Z boson mass within ± 15 GeV. CMS defines H_T to be the scalar sum of all jets' p_T whose angular separation to the nearest lepton is $\Delta R > 0.4$.

We have generated CKKW-matched [30] $t\bar{t} + W^\pm/Z$, $W^\pm W^\pm jj$ and $W^\pm Zjj$, which constitute the dominant backgrounds using SHERPA [31]. The QCD corrections to these processes are known to be small [20,21,32,33]. The signal events are produced with MADGRAPH/MADEVENT V5 [34] using a FEYNRULES [35] interface to our model implementation described in Ref. [8].¹ The signal events are subsequently showered and hadronized with HERWIG++ [36]. In the analysis we include Gaussian detector smearing of the jets and leptons on the basis of Ref. [37]:

$$\begin{aligned} \text{jets: } \frac{\Delta E}{E} &= \frac{5.2}{E} \oplus \frac{0.16}{\sqrt{E}} \oplus 0.033, \\ \text{leptons: } \frac{\Delta E}{E} &= 0.02, \end{aligned} \quad (12)$$

and we include the missing energy response from recent particle flow fits of CMS [38] via the fitted function [39]²

$$\text{missing energy: } \frac{\Delta E_T^{\text{miss}}}{E_T^{\text{miss}}} = \frac{2.92}{E_T^{\text{miss}}} - 0.07. \quad (13)$$

The jet resolution parameters can be improved by particle flow too; we, however, choose the more conservative parametrization to capture the effect of an increased jet energy scale uncertainty in the forward detector region, which especially impacts the WBF-like selection.

We use the background samples to generate an efficiency profile over the 8 CMS search regions [cf. Fig. 2(a)] we are focusing on

¹We note that the $2j + E_T^{\text{miss}} + \ell^+ \ell^-$ signal also receives contributions from the vertices $H_0 W^\pm W^\pm$, $H'_0 W^\pm W^\pm$ and $H_5^0 W^\pm W^\pm$, which are present in diagrams containing an internal t -channel neutral Higgs boson connecting the W^\pm 's emitted from the two quark lines. These diagrams are important to ensure unitarity in longitudinal weak boson scattering for high energy ($H_5^{\pm\pm}$ off-shell) scattering. We have checked that their numerical contribution is negligible in the $H_5^{\pm\pm}$ resonant region captured by Fig. 1, and therefore we have not included them explicitly in this work.

²The missing energy response might vary from Ref. [38] when the analysis is performed by the experiments. This is clearly beyond the scope of this work, but we believe that our parametrization is well justified for demonstration purposes.

region 1: high p_T , $H_T > 80$ GeV, $E_T^{\text{miss}} > 120$ GeV,
region 2: low p_T , $H_T > 200$ GeV, $E_T^{\text{miss}} > 120$ GeV,
region 3: high p_T , $H_T > 200$ GeV, $E_T^{\text{miss}} > 120$ GeV,
region 4: low p_T , $H_T > 450$ GeV, $E_T^{\text{miss}} > 50$ GeV,
region 5: high p_T , $H_T > 450$ GeV, $E_T^{\text{miss}} > 50$ GeV,
region 6: low p_T , $H_T > 450$ GeV, $E_T^{\text{miss}} > 120$ GeV,
region 7: high p_T , $H_T > 450$ GeV, $E_T^{\text{miss}} > 120$ GeV,
region 8: high p_T , $H_T > 450$ GeV, $E_T^{\text{miss}} > 0$ GeV, (14)

which we apply to our signal hypothesis.³ To obtain CLS exclusion limits [40] we perform a log likelihood hypothesis test as described in [41], where we marginalize over the background uncertainty quoted in [25] [and indicated in Fig. 2(a)].

The result is shown in Fig. 2(b), where we plot the observed and expected 95% confidence level constraints on the signal strength

$$\xi = \frac{\sigma(H^{\pm\pm} jj) \times \text{BR}(H^{\pm\pm} \rightarrow W^\pm W^\pm \rightarrow \text{leptons})}{[\sigma(H^{\pm\pm} jj) \times \text{BR}(H^{\pm\pm} \rightarrow W^\pm W^\pm \rightarrow \text{leptons})]_{\text{ref}}} \quad (15)$$

as a function of the mass of $H^{\pm\pm}$. Since the total width is dominated by $H_5^{\pm\pm} \rightarrow W^\pm W^\pm$, we have $\xi \simeq s_H^2$. ξ sets a limit in reference to a point that we choose with values

³Since the CMS analysis does not tag on the number of jets, we have also considered production modes with same-sign dilepton and E_T^{miss} , but where more than 2 jets are produced. In a model with an extended Higgs sector, the cross section to produce such final states could potentially be very different from the SM rate. We have explicitly checked that production rates for $pp \rightarrow E_T^{\text{miss}} + \ell^\pm \ell^\pm + (>2j)$ when extra states are included are negligible with respect to the main contribution to the signal, i.e. $pp \rightarrow W^\pm W^\pm jj$, with an s -channel exchanged $H_5^{\pm\pm}$, is the dominant process. To establish this, we have computed the impact of $pp \rightarrow Z \rightarrow H^{\pm\pm} H^{\mp\mp} \rightarrow W^\pm W^\pm jjjj$, $pp(\rightarrow W^\pm) \rightarrow H^{\pm\pm} H_{3,5}^\pm \rightarrow W^\pm W^\pm jjjj$, $pp \rightarrow H^{\pm\pm} \rightarrow W^\pm H_{3,5}^\pm \rightarrow W^\pm W^\pm jj$ and $gg \rightarrow H_0 \rightarrow H^{\pm\pm} H^{\mp\mp} \rightarrow W^\pm W^\pm jjjj$ ($gg \rightarrow H_3^0 \rightarrow H^{\pm\pm} H^{\mp\mp}$ would also be possible, but $H_3^0 \rightarrow H^{\pm\pm} H^{\mp\mp}$ is forbidden) to the signal estimate, and found negligible contributions. More precisely, the only process that could have a marginal impact is $gg \rightarrow H_0 \rightarrow H^{\pm\pm} H^{\mp\mp} \rightarrow W^\pm W^\pm jjjj$, when $m_{H_0} > 2m_{H_5^{\pm\pm}}$. While H_0 can be heavier than the quintet, the situation where it is heavy enough to have an open two-body decay channel into a quintet pair is not very frequent. For example we have checked that this is the case by inspecting the points we considered in our previous study [8]. In the present work, only for the template scenarios with light quintets ($m_{H_5} < 250$ GeV) have we found that this is possible, and in such cases we have checked that the total contribution from this subprocess can enhance the signal by a factor of 1.5. This is not enough to change our estimates significantly. We are therefore confident that the approximations we are using for the simulation of signal and backgrounds are robust. We note, however, that the contributions discussed in this footnote are model dependent because they explicitly probe the larger particle content and the Higgs interactions due to the potential.

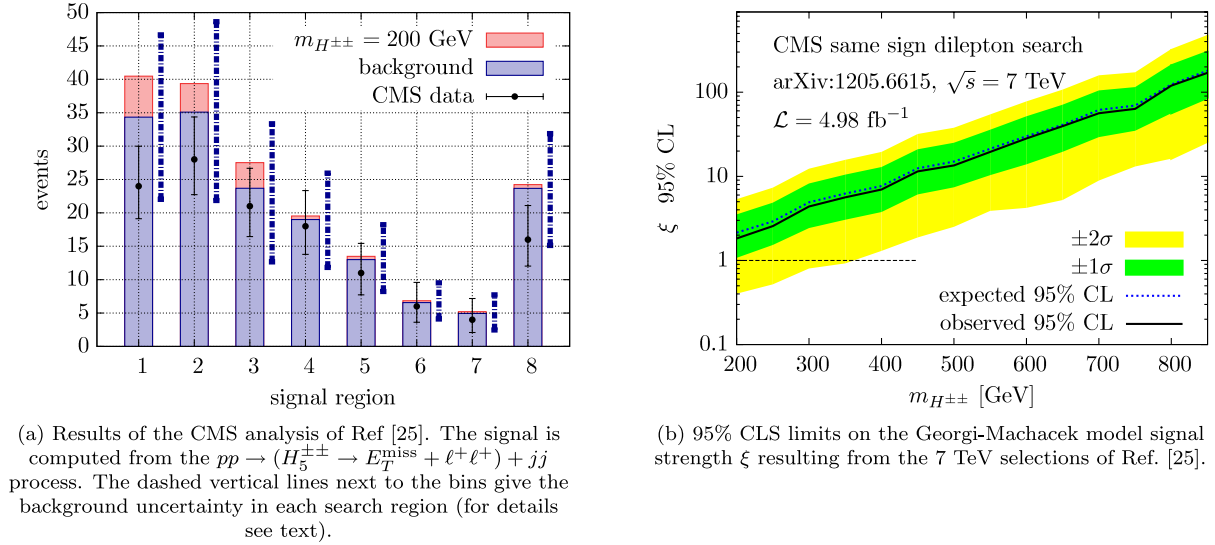


FIG. 2 (color online). Estimated signal and background events, measured data and corresponding CLS limits for the 7 TeV CMS selection of Ref. [25].

$$s_H = 1/\sqrt{2}, \quad m_3 = 500 \text{ GeV} \quad (16)$$

for the Higgs mixing and triplet mass, i.e. a hW^+W^- -like value of the $H^{\pm\pm}W^\pm W^\pm$ coupling. These are also values allowed by constraints from nonoblique corrections, in particular due to $Z \rightarrow b\bar{b}$ measurements [42]. All other parameters are chosen such that the 125 GeV Higgs state has a coupling to weak gauge bosons that agrees with the SM within 5%. Given the large triplet Higgs vev, this is an optimistic scenario, but we stress that it only serves to establish a baseline for the measurement of ξ .

We see that the CMS analysis, which cuts on H_T , i.e. central jet activity instead of WBF-type topologies, only starts to probe the model for $H_5^{\pm\pm}$ masses close to the W^\pm threshold. The discriminative power always predominantly comes from the search region 1, which is closest to a typical WBF selection among the eight search channels of Eq. (14). As we will see in Sec. V, once other constraints such as electroweak precision measurements and direct Higgs search constraints are included, the SUSY search does not provide a strong constraint on the parameter space of the Georgi-Machacek model.

As can be guessed from Fig. 2(a), excluding the triplet via the CMS SUSY search is hampered by the large systematic uncertainties. If we omit the systematic uncertainties and compute the excluded signal strength only on the basis of statistical uncertainties, the CMS analysis excludes $\xi = 0.68$ for $m_{H_5^{\pm\pm}} = 200$ GeV. This enables a qualitative projection of the situation when the 8 TeV sample is included. Because of the larger data sample we can expect that the background uncertainty is reduced by a larger available set of subsidiary background measurements at higher statistics. CMS has an 8 TeV data sample of $\mathcal{L} \approx 23 \text{ fb}^{-1}$. With this sample and a systematic

uncertainty reduced by 50%, CMS starts probing the triplet parameter space for $H_5^{\pm\pm}$ masses up to $m_{H_5^{\pm\pm}} \approx 250$ GeV.

A. Toward a more WBF-like selection

We modify the above analysis toward a more signal-like selection. The base cuts are identical, but this time we extend the jet clustering over the full hadronic calorimeter range $|\eta| < 4.5$ and add standard WBF cuts via

$$m_{j_1 j_2} > 500 \text{ GeV} \quad \text{and} \quad |y_{j_1} - y_{j_2}| > 4. \quad (17)$$

This means that instead of exclusively clustering central jets, we also allow more forward jets, so the systematic uncertainties might be different compared to the CMS analysis we discussed in Sec. III. The fake background contribution, in particular, can quantitatively only be assessed by the experiments themselves. To get a qualitative estimate, we simulate $W + \text{heavy flavor events}$ ⁴ that we match onto the CMS analysis region 1 and use a flat extrapolation to the signal region described above. This yields approximately an estimate of the background composition of again $\sim 60:40$ of fake:irreducible. To calculate confidence levels we assume a systematic uncertainty on the background of 75% (which is a rather conservative estimate in light of the CMS search of the previous section). As expected, the WBF selection reduces the background without degrading the signal too much, therefore enhancing the signal vs background ratio. The expected exclusion limit on the basis of these parameters is shown in Fig. 3. We see that already with the 4.98 fb^{-1} data set we can expect limits on the model up to masses $m_{H^{\pm\pm}} \approx 300$ GeV. If the background uncertainty is

⁴Following Ref. [25] this is expected to be the dominant contribution of the fake background.

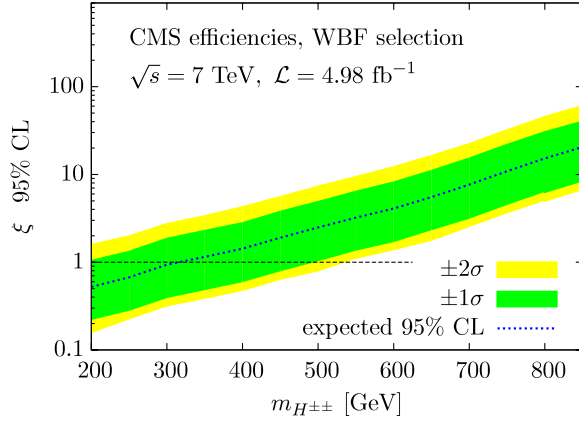


FIG. 3 (color online). Expected exclusion limits for a more WBF-like analysis based on Ref. [25]. For details see text.

reduced by 50%, the full 8 TeV data set probes triplet models up to masses $m_{H^{\pm\pm}} \simeq 420$ GeV for our reference value $s_H = 1/\sqrt{2}$.

IV. PROSPECTIVE SENSITIVITY AND DISCOVERY THRESHOLDS AT 14 TeV

Switching to higher center-of-mass energy changes the sensitivity to the model dramatically. WBF-like cross sections increase by a factor of ~ 5 when doubling the available center-of-mass energy from 7 TeV to 14 TeV [43]. We can therefore introduce additional WBF criteria like a central jet veto to further suppress the QCD backgrounds, as well as lepton vetoes to remove the $WZjj$ backgrounds.

Our event generation for the 14 TeV analysis follows the 7 TeV tool chain. We use the anti- k_T jets with $R = 0.5$, and lower the p_T thresholds to 20 GeV in $|\eta_j| < 4.5$. We enlarge the requirement on the tagging jets invariant mass to $m_{jj} > 600$ GeV and furthermore require that the jets fall in opposite detector hemispheres $y_{j_1} \cdot y_{j_2} \leq 0$. The leptons are required to be isolated from the jets by a

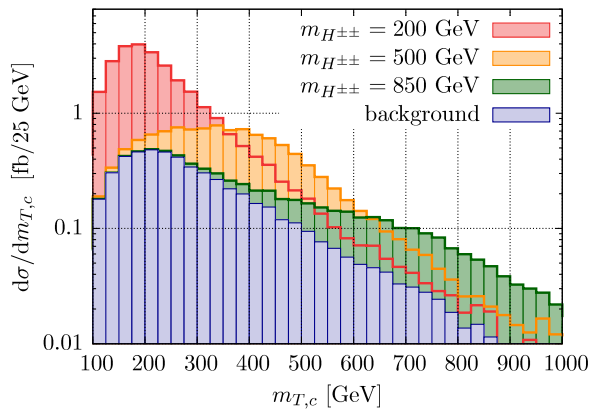
distance $\Delta R_{\ell j} = 0.4$. This time we veto events with a third lepton and a central jet that meets the above requirement. No restrictions on E_T^{miss} are imposed. The result is a signal-dominated selection, which not only allows us to highly constrain s_H over a wide range of $H_5^{\pm\pm}$ masses but also enables the approximate reconstruction of the $H_5^{\pm\pm}$ mass from a Jacobian peak in the transverse cluster mass distribution,

$$m_{T,c}^2 = \left(\sqrt{(p_{\ell_1} + p_{\ell_2})^2 + |\vec{p}_{T,\ell_1} + \vec{p}_{T,\ell_2}|^2 + E_T^{\text{miss}})^2} - |\vec{p}_{T,\ell_1} + \vec{p}_{T,\ell_2} + \vec{E}_T^{\text{miss}}|^2 \right), \quad (18)$$

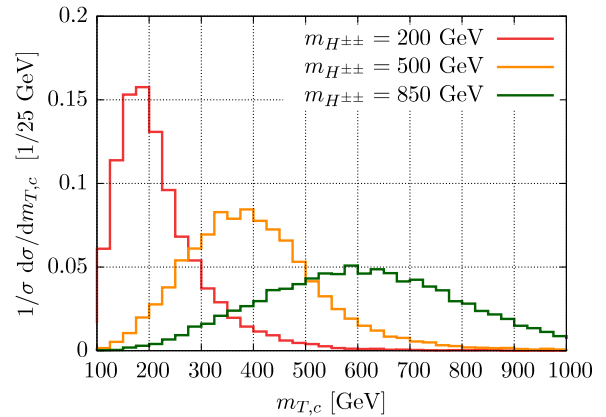
in case such a model is realized in nature. Because of detector resolution effects, missing energy uncertainty and initial state radiation, the mass resolution of the Jacobian peak degrades significantly when considering heavier $H^{\pm\pm}$ masses, as shown in Fig. 4. A statistically significant measurement will still be possible; the mass parameter determination, however, will be poor.

The above event selection serves two purposes. First, all QCD-induced backgrounds (which are characterized by central jet activity at moderate m_{jj}) are highly suppressed. We suppress the backgrounds further by imposing lepton and central jet vetoes. Note that this also removes signal contributions that arise from processes other than WBF. As a result we directly constrain s_H . After all cuts have been applied, the irreducible background is completely dominated by the electroweak SM $pp \rightarrow (W^\pm W^\pm \rightarrow \ell^\pm \ell^\pm + E_T^{\text{miss}})jj$ contribution at $\mathcal{O}(\alpha^6 \alpha_s^0)$. This background is comparably small and under good perturbative control [20].

The fake background contribution can quantitatively only be assessed by the experiments themselves. To get a qualitative estimate, we again simulate W + heavy flavor events that we match onto the CMS analysis region 1 and use a flat extrapolation to the 14 TeV WBF selection described above as already done for the 7 TeV WBF selection criteria. This yields approximately an estimate



(a) Transverse cluster mass distribution.



(b) Normalized signal cluster mass distribution.

FIG. 4 (color online). Transverse cluster mass distribution for signal + background of the $H_5^{\pm\pm}$ search as discussed in Sec. IV.

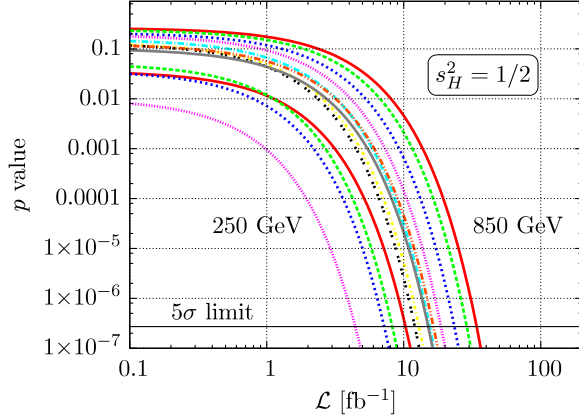


FIG. 5 (color online). Associated p values for a search based on the single discriminant $m_{T,c}$ as a function of $m_{H^{\pm\pm}}$ and the integrated luminosity. The curves, moving from left to right, correspond to $H_5^{\pm\pm}$ masses between 250 GeV and 850 GeV in steps of 50 GeV.

of background composition of 50:50 of fake:irreducible. We furthermore assume a systematic shape uncertainty of the background of 35% (flat), which follows from 10% and 25% uncertainties on the irreducible and fake backgrounds, respectively.

In Fig. 5 we show the associated p values for a search based on the observable $m_{T,c}$ of Fig. 4 for the reference point $s_H = 1/\sqrt{2}$. The signal cross section scales with $s_H^2 \sim s_H^{\text{ref}2} \sqrt{\mathcal{L}/\mathcal{L}^{\text{ref}}}$. In principle this implies that the LHC provides us enough sensitivity for discoveries down to $s_H^2 \sim 0.1$ for heavy masses for the considered $H_5^{\pm\pm}$ mass range.

In Fig. 6 we show the expected 95% confidence level constraints as a function of the $H_5^{\pm\pm}$ mass for luminosities

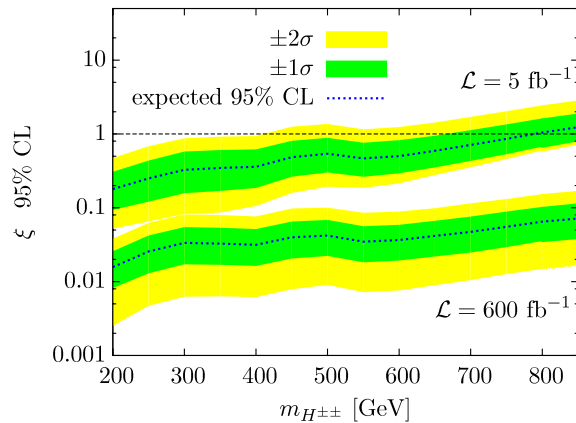


FIG. 6 (color online). Associated signal strength limits at 95% confidence level computed from a binned log likelihood hypothesis test on the basis of the single discriminant $m_{T,c}$, Fig. 4(a), using the CL_s method [40]. We show results for two luminosity values for running at 14 TeV center-of-mass energy, 5 fb^{-1} and 600 fb^{-1} .

5 fb^{-1} and 600 fb^{-1} . The expected constraint on the signal strength ξ can directly be interpreted as a limit on the $H_5^{\pm\pm} W^+ W^-$ coupling Eq. (11). As can be seen from this figure, an analysis based on the WBF channel is a very sensitive search, eventually yielding constraints $s_H^2 \lesssim 0.05$ over the entire parameter range. On the one hand, since $s_H \ll 1$ is required by the W/Z mass ratio in a complex triplet extension, the expected constraint is not good enough to constrain the entire parameter space. On the other hand, it is possible to constrain the bulk of the parameter space in the context of the Georgi-Machacek model, which typically allows larger values for s_H [8].

V. COMBINING DIRECT $H_5^{\pm\pm}$ SEARCHES WITH OTHER CONSTRAINTS

In this section we want to compare the exclusion potentials due to searches for a doubly charged scalar obtained in the previous sections with representative points for the parameter space still allowed for the GM model. In particular, in a previous study [8], we have shown that this space is large enough to accommodate both the case where the 125 GeV Higgs boson has an enhanced $\gamma\gamma$ decay rate with respect to the SM value and the case where the couplings for the Higgs boson candidate are SM-like.

It is therefore natural to study whether the (future, possible) nonobservation of excesses in searches for doubly charged states has the potential to completely rule out these two scenarios, and hence the GM extension of the Higgs sector.

Before showing the results, we summarize the information included in the two sets of points we will use in the following. We list here only the aspects that are relevant for the present work, and we refer the reader to Ref. [8] for a detailed explanation of how these results were obtained:

Direct ATLAS, CMS: The points that we consider correspond to scenarios where the H'_0 scalar is the observed Higgs boson. Therefore we restrict to the case where the other singlet H_0 is heavier, and we require that neither H_0 nor H_3^0 violate the LHC exclusion limits on scalar production. This case has been discussed in Ref. [8] in detail.

Consistency with 125 GeV signal: We require that the tree-level couplings of H'_0 with fermions and gauge bosons, and the loop-induced coupling with gluons, are such that H'_0 reproduce the observed total signal strength as well as the individual signal strengths for WW ($\xi_{H \rightarrow WW}$) and $\gamma\gamma$ ($\xi_{H \rightarrow \gamma\gamma}$) decays. In particular, at this level we distinguish between a scenario where we have room to reproduce an excess in the photonic branching ratio and another where signal strengths agree with the SM values within 20%. For further details on the scan we refer the reader to Ref. [8].

Oblique corrections: In our previous study we have also taken into account constraints from electroweak precision measurements. In particular we studied both cases where the T parameter is used or not, since at one loop the radiative corrections are not unambiguously defined.

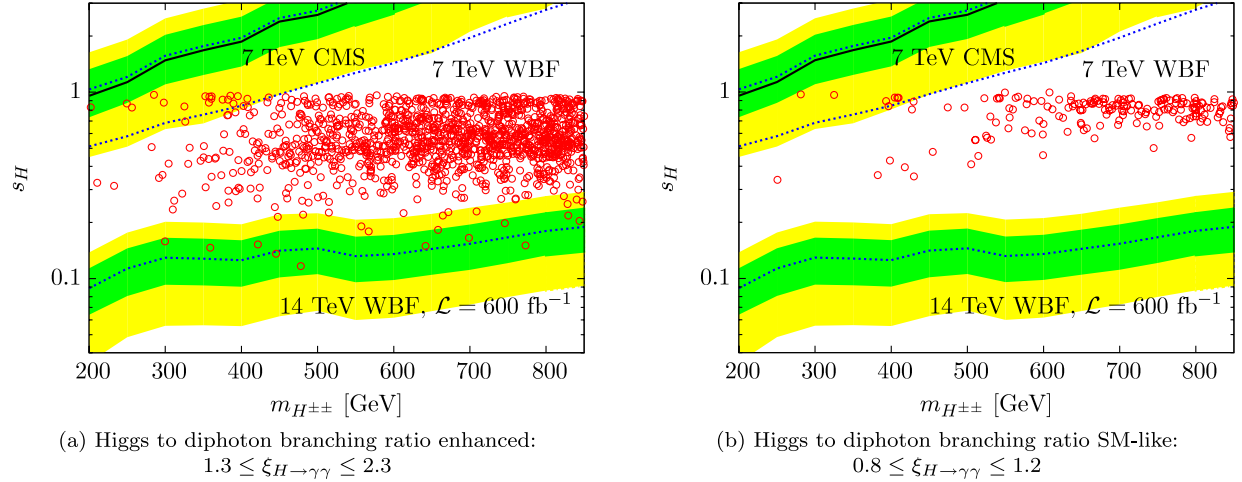


FIG. 7 (color online). Exclusion yield of the searches described in the previous sections when included in a model scan over the Georgi-Machacek model. The parameter points are consistent with electroweak precision measurements, current direct LHC and LEP constraints, and reproduce the signal strength of the measured Higgs boson in the observed weak boson decay channels $H \rightarrow W^+ W^-$, ZZ . The dotted contours represent the expected exclusion, the green and yellow regions reflect the ± 1 , ± 2 sigma uncertainty bands. The contours are, from top to bottom, the eight channel CMS SUSY search described in Sec. III, the adapted 7 TeV WBF search described in Sec. III A (both 4.98 fb^{-1} luminosity) and the fully differential search at 14 TeV center-of-mass energy of Sec. IV (600 fb^{-1} luminosity).

In this work we have decided not to consider this subtle but important issue, which we instead discussed at length in Ref. [8]: therefore we used the sets of points labeled in our previous paper as “S. param included,” i.e. the results obtained here are independent of any T parameter constraint or fine-tuning [27].

Nonoblique corrections ($Zb\bar{b}$): In our previous work we have not explicitly included constraints due to the fermionic coupling of the custodial-triplet charged states H_3^\pm . The presence of these states might change significantly several observables involving b quarks, because of possibly large values for the $H_3^+ tb$ coupling. One of the more important observables to look at is R_b , defined as $\Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$. Changes in the SM value prediction of R_b induced by the GM model have been computed in Ref. [42]. We have reproduced these results and checked that a large portion of the points we will use in the following, which were considered still allowed in our previous paper, survive also the bounds from $Z \rightarrow b\bar{b}$.⁵

As the above discussion shows, in our previous study we have taken into account essentially all the available constraints from direct and indirect searches. In particular, for

this paper we also checked that the conclusions we reached in Ref. [8] remain essentially unchanged also when non-oblique corrections (in the $Z \rightarrow b\bar{b}$ case) are included.

In Figs. 7(a) and 7(b) we show the exclusion potential of the search strategies discussed in Secs. III and IV, together with the surviving points for the two scenarios we just described. The standard color coding is used for the exclusion plots, which here are shown as a function of s_H and $m_{H^{\pm\pm}}$. From these plots we conclude that the searches at 7 TeV, if extended with WBF-like selection cuts, start to be able to probe, and hence exclude, some of the surviving scenarios. The more relevant result, however, is that WBF searches on the 14 TeV data will have the potential to completely rule out all the points that survive all other constraints. This search has therefore the potential to become a decisive obstacle that models with Higgs triplets and large triplet-doublet mixing have to pass in order not to be excluded. As such, it would be very important for LHC experimental collaborations to look into these final states. In particular an analysis based on the same-sign lepton WBF channel serves to also constrain the parameter region that is allowed in other recent analyses such as Ref. [14].

VI. SUMMARY

Higgs triplets as implemented in the Georgi-Machacek model provide a viable extension of the SM Higgs sector, which can be efficiently probed at the LHC. We have demonstrated that while current analyses of same-sign lepton final states do not provide a strong enough constraint on the presence of doubly charged scalar bosons

⁵For the sake of completeness, we would like to point out that recent results in the computation of two-loop corrections for the SM $Zb\bar{b}$ coupling lead to sizable effects that have not been taken into account in previous literature [44]. Including these effects, however, goes beyond the purpose of this study, although it could be potentially relevant for constraints only due to nonoblique corrections. We will, however, show that searches for WBF-produced doubly charged states are very powerful as exclusion tests for these models, and therefore our main results will hold, regardless of the relative size of these loop effects.

decaying to same-sign W 's on the basis of SUSY searches, the enlarged statistical sample of the 8 TeV 2012 run should start constraining this model via the nonadapted SUSY search strategy. Furthermore we have shown that a simple modification of these SUSY searches allows us to constrain the model already with 7 TeV data even for a conservative background estimate. The model can be ultimately verified or ruled out at the LHC with 14 TeV in a clean WBF selection.

Our results are quite general and at the same time realistic as far as models with triplets are considered. In particular, studying $H^{\pm\pm} \rightarrow W^{\pm}W^{\pm}$ rather than the more commonly considered case $H^{\pm\pm} \rightarrow \ell^{\pm}\ell^{\pm}$ seems to be more natural, because of the dominance of the former decay over the latter for the bulk of the parameter space

independent of the considered triplet scenario. Moreover, the analysis strategies we studied in this work are quite standard, but at the same time can lead to conclusive results for a complete exclusion of Higgs sectors with triplets. We therefore think that it would be very important for the LHC experimental collaborations to consider these searches in addition to the already considered and simpler case $H^{\pm\pm} \rightarrow \ell^{\pm}\ell^{\pm}$.

ACKNOWLEDGMENTS

C.E. acknowledges funding by the Durham International Junior Research Fellowship scheme. We also acknowledge Chris Hays for interesting discussions and for comments on the draft.

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